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D1-brane in β-deformed background

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Abstract: We study various configurations of rotating and wound D1-brane in $AdS_5 \times S^5$ background and in its β deformed version. We find giant magnon and spike solutions on the world-volume of D1-brane in $\mathrm{AdS}_5\times$ S^5 background. We also analyse the equations of motion of D1-brane in β-deformed background. We show that in the limit of large electric flux on world-volume of D1-brane they reduce to the equations that describe collection of large number of fundamental strings. We also construct rotating and wound D1-brane solution that has two equal spins on S^5_γ.

Keywords: AdS-CFT Correspondence, D-branes.

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Contents

1. Introduction

String theory on $\mathrm{AdS}_5\times$ S^5 should be dual to $\mathcal{N}$ =4 supersymmetric Yang-Mills (SYM) theory in four dimensions [1]. This duality conjecture has undergone varieties of tests at various levels. One of them is the spectrum matching at the both sides of the duality. It turns out to be very difficult to present the full spectrum of the theory and then to compare it with spectrum of the anomalous dimension of the gauge theory operators. Hence it is natural to examine various limits of the duality conjecture. One such interesting class of operators is that which carry large charges, such as large angular momentum [2]. In this sector one can use the semiclassical approximation to find the energy spectrum. In the gauge theory one needs trace of very long operators. It further was analysed in a beautiful paper [3] that the Hamiltonian of a Heisenberg's spin chain system is related with that of the dilatation operator in N=4 Supersymmetric Yang-Mills theory.[1] Since then lot of work

[1]For recent reviews, see [4 – 6].

has gone in to understand the interplay between the integrability of the string theory on AdS, and making connection with more than handful of gauge theory operators.

One such low lying spin-chain system corresponds to magnon like excitation. Hofman and Maldacena [7] have been able to match these magnon excitations to that of a class of semiclassical rotating string state on $R \times S^2$.[2] They move around the equator of the sphere and have large angular momentum and energy. The giant magnon solution of correspond to operators where one of the SO(6) charge, J, is taken to infinity, keeping the $E - J$ fixed ($J \rightarrow \infty, \lambda = \text{fixed}, p = \text{fixed}$). These spin chain giant magnon excitations satisfy a dispersion relation of the type (in the large ' t Hooft limit (λ))

$$E - J = \frac{\sqrt{\lambda}}{\pi} \left| \sin \frac{p}{2} \right| , \tag{1.1}$$

where p is the magnon momentum, which on the string theory side correspond to a deficit angle ϕ.

Another class of solutions with spike like configuration in the $\text{AdS}_5 \times \text{S}^5$ has also been found out [25, 26] and in the gauge theory side these correspond to single trace operators with large number of derivatives. More recently in [27] is has been analysed that for an infinitely wound string around S^2 and S^3 with a single spike can be obtained from a general solution of rigidly rotating string on the sphere. More interestingly in a certain parameter space the solutions can also be thought of as giant magnon. In fact both the giant magnon and single spike solutions has been obtained by taking different limits on a general class of rotating solutions. So it seems natural that single spike solutions fall into the same class of solutions as that of giant magnon. The difference is that the spike solutions do not correspond directly to that of any gauge theory operators. For these single spike solution, like the magnon dispersion relation can be summarized by

$$E - T\Delta\psi = \frac{\sqrt{\lambda}}{\pi} \bar{\theta} , \tag{1.2}$$

where $\Delta\psi$ is the difference in the angle between two spikes.

It is very interesting knowing the results of the elementary string wound around $\text{AdS}_5 \times \text{S}^5$ has in its solution both magnon and spike like configuration, what happens in case of D-string ? It will enhance our understanding beyond the elementary string configurations, and might also give generalization of these as well. We would like to analyse this possibility in the present paper. For our purpose we will restrict ourselves to the less supersymmetric Lunin-Maldacena background [30]. This background has been conjectured to the Leigh-Strassler marginal deformation of $\mathcal{N} = 4$ SYM. The study of properties of classical string in these backgrounds was performed in many papers, for example [20, 27, 32 − 46]. Our goal is to perform similar analysis in the case of D1-brane. First of all, the D-brane unlike the fundamental string couple to the dilaton explicitly. So it is rather interesting to see if similar configurations of giant magnon and single spike solutions exists in case of D-branes. The corresponding operators in the dual gauge theory is still unknown. However the existence of these semiclassical states in string theory side might give a hint that there

[2] For some related papers, see [8 − 24, 28, 29, 36].

might be similar states in a gauge theory. We will solve the equations coming from the Dirac-Born-Infeld action on the D1-string, and will analyse the possibility of getting the giant magnon and single spike like solutions on its worldvolume.

The rest of the paper is organized as follows. In section two for notational details we first write down the DBI action of a D1-string and derive the equations of motion for the worldvolume coordinates and write down the background fields corresponding to β deformed AdS$_5\times$ S^5.[3] We further derive the equations of motion of a rotating D1-brane in this background and write down its equations of motion. Section three is devoted to the study of the solutions of the equations of motion derived in section two, and find out solutions that correspond to spike and giant magnon is case of S^2 and S^3 embedded inside S^5. We show the existence of the similar dispersion relation, as in case of magnon solutions on fundamental strings, in the presence of worldvolume gauge field. In section four we discuss the possibility of finding out the magnon and single spike solution in case of D1-brane in β deformed AdS$_5\times$ S^5 background. we find more general solutions and show that in the limit of large electric flux on world-volume of D1-brane they reduce to the equations that describe collection of large number of fundamental strings. We also construct rotating and wound D1-brane solution that has two equal momenta on S^5_γ. In section five we present our conclusions. Finally, some details of the calculations are summarized in the appendices.

2. D1-brane in β-deformed background

The dynamics of D1-brane in general background is governed by following action

$$
\begin{aligned}
S &= S_{DBI} + S_{WZ}\,,\\
S_{DBI} &= -\tau_1 \int d^2\xi e^{-\Phi}\sqrt{-\det \mathbf{A}}\,,\\
\mathbf{A}_{\alpha\beta} &= \partial_\alpha x^M \partial_\beta x^N G_{MN} + (2\pi\alpha')\mathcal{F}_{\alpha\beta}\,,\\
\mathcal{F}_{\alpha\beta} &= \partial_\alpha A_\beta - \partial_\beta A_\alpha - (2\pi\alpha')^{-1} B_{MN}\partial_\alpha x^M \partial_\beta x^N\,,\\
S_{WZ} &= \tau_1 \int e^{(2\pi\alpha')\mathcal{F}} \wedge C\,,
\end{aligned}
\tag{2.1}
$$

where τ_1 is D1-brane tension, $\xi^\alpha, \alpha = 0,1$ are world-volume coordinates and where A_α is gauge field living on the world-volume of D1-brane. Note also that C in the last line in (2.1) means collection of Ramond-Ramond fields. The equations of motion derived from this action has been summarized in appendix-A.

Our goal is to study dynamics of D1-brane in β-deformed $AdS_5 \times S^5$ background [30]. Let us now review its main properties.

The β-deformed $AdS_5 \times S^5$ background can be obtained from pure $AdS_5 \times S^5$ by a series of TsT transformations as was shown in [45]. The deformation parameter $\beta = \gamma + i\sigma_d$ is in general a complex number however we restrict to the case when $\sigma_d = 0$, where the corresponding deformation is called γ deformation. The resulting supergravity background

[3]For review of integrable deformations, see [31].

takes the form

$$ds^2 = R^2\left(ds^2_{AdS_5} + \sum_{i=1}^{3}(d\mu_i^2 + G\mu_i^2 d\phi_i^2) + \tilde{\gamma}^2 G\mu_1^2\mu_2^2\mu_3^2(\sum_{i=1}^{3} d\phi_i^2)\right) . \tag{2.2}$$

It is important to note that this background also contains in addition a non-trivial dilaton field as well as RR and NS-NS form fields:

$$\begin{aligned}
B &= R^2\tilde{\gamma}G(\mu_1^2\mu_2^2 d\phi_1 d\phi_2 + \mu_2^2\mu_3^2 d\phi_2 d\phi_3 + \mu_1^2\mu_3^2 d\phi_1 d\phi_3)\,, \\
e^{2\Phi} &= e^{2\Phi_0}G\,, \quad G = \frac{1}{1+\tilde{\gamma}^2(\mu_1^2\mu_2^2 + \mu_2^2\mu_3^2 + \mu_1^2\mu_3^2)}\,, \\
C_2 &= -R^2\tilde{\gamma}e^{-\Phi_0}\omega_1 d\psi\,, \quad d\omega_1 = 12\cos\theta\sin^3\theta\sin\psi\cos\psi d\theta\wedge d\psi\,, \\
F_5 &= dC_4 = 4R^4 e^{-\Phi_0}(\omega_{AdS_5} + \omega_{S^5})\,, \\
\mu_1 &= \sin\theta\cos\psi\,, \quad \mu_2 = \cos\theta\,, \quad \mu_3 = \sin\theta\sin\psi\,,
\end{aligned} \tag{2.3}$$

where $(\theta, \psi, \phi_1, \phi_2, \phi_3)$ are the usual S^5 variables and where $\omega_{AdS_5}, \omega_{S^5}$ are corresponding volume forms of AdS_5 and S^5 respectively. Finally $\tilde{\gamma}$ is defined as

$$\tilde{\gamma} = R^2\gamma\,, \quad R^2 = \sqrt{4\pi\alpha'^2 e^{\Phi_0}N} . \tag{2.4}$$

Our goal is to study spikes solutions on D1-brane that moves on S^3_γ.[4] We represent this space as a subspace of γ-deformed $AdS_5\times S^5$ presented above

$$\mu_3 = 0\,, \quad \phi_3 = 0 \tag{2.5}$$

or equivalently

$$\psi = 0\,, \quad \phi_3 = 0 . \tag{2.6}$$

The relevant part of the γ deformed $AdS_5\times S^5$ is

$$ds^2 = R^2(-dt^2 + d\theta^2 + G\sin^2\theta d\phi_1^2 + G\cos^2\theta d\phi_2^2) \tag{2.7}$$

and the dilaton, RR and NS-NS two-forms take the form

$$\begin{aligned}
B_{\phi_1\phi_2} &= R^2\tilde{\gamma}G\sin^2\theta\cos^2\theta\,, \quad e^{2\Phi} = e^{2\Phi_0}G\,, \\
G &= \frac{1}{1+\tilde{\gamma}^2\sin^2\theta\cos^2\theta} .
\end{aligned} \tag{2.8}$$

Note that due to the fact that C_2 vanishes we do not need to worry about the Wess-Zumino term of D1-brane effective action.

Let us now consider following ansatz

$$t = \kappa\tau\,, \quad \theta = \theta(y)\,, \quad \phi_1 = \omega_1\tau + \tilde{\phi}_1(y)\,, \quad \phi_2 = \omega_2\tau + \tilde{\phi}_2(y)\,, \tag{2.9}$$

where we have defined the variable y as

$$y \equiv \alpha\sigma + \beta\tau . \tag{2.10}$$

[4] various supersymmetric D-brane embeddings in the beta deformed background was studied in [48]. However our aim is different in this paper

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Now we start to analyze the equations of motion (A.1) and (A.4) with the ansatz above. The explicit form of the matrix A is summarized in (A.6) - (A.12).

Let us see the equations of motion (A.1). Firstly, for $x^0 \equiv t$ this equation implies an existence of conserved quantity

$$-\frac{e^{-\Phi_0}R^2\alpha^2}{\sqrt{G}\sqrt{-\det \mathbf{A}}}(G\sin^2\theta\omega_1\tilde{\phi}_1' + G\cos^2\theta\omega_2\tilde{\phi}_2') = A\,, A = \text{const}\ . \tag{2.11}$$

In what follows we presume that $A < 0$. Further, the equations of motion for ϕ_1 takes the form

$$-A\frac{R^2\sin^2\theta}{G}\frac{[\omega_2 G\cos^2\theta(\tilde{\phi}_1'\omega_2 - \tilde{\phi}_2'\omega_1) - \phi_1'\kappa^2]}{\sin^2\theta\omega_1\tilde{\phi}_1' + \cos^2\theta\omega_2\tilde{\phi}_2'} + \sin^2\theta\cos^2\theta\omega_2 R^2\tilde{\gamma}G\Pi = B\,, \tag{2.12}$$

where $B = \text{const}$. Π is a constant that counts the number of fundamental strings stretched along the world volume of D1-brane defined in (A.9).

In the same way the equation of motion for ϕ_2 gives

$$-\frac{AR^2\cos^2\theta}{G}\frac{[\omega_1 G\sin^2\theta(\tilde{\phi}_2'\omega_1 - \tilde{\phi}_1'\omega_2) - \tilde{\phi}_2'\kappa^2]}{\sin^2\theta\omega_1\tilde{\phi}_1' + \cos^2\theta\omega_2\tilde{\phi}_2'} - \sin^2\theta\cos^2\theta\omega_1 R^2\tilde{\gamma}G\Pi = C\,, \tag{2.13}$$

where again C is constant. Before we proceed to explicit solutions of these equations we determine conserved quantities that reflect isometries of given background. These currents are conserved as a consequence of the equation of motion:

$$\partial_\alpha \mathcal{J}^\alpha_{t,1,2} = 0\ . \tag{2.14}$$

Then corresponding conserved charges take the form

$$P_t = \int_0^{2\pi} d\sigma \mathcal{J}_t^\tau\,, \quad J_1 = \int_0^{2\pi} d\sigma \mathcal{J}_1^\tau\,, \quad J_2 = \int_0^{2\pi} d\sigma \mathcal{J}_2^\tau\,, \tag{2.15}$$

where we presume that D1-brane has compact support and also that world-volume fields obey periodic boundary conditions.

After the general discussion of the properties of D1-brane in β-deformed background we proceed to the study of the solutions of the corresponding equations of motion.

3. D1-brane in $AdS_5 \times S^5$

We begin our discussion with the study of the dynamics of D1-brane in original $AdS_5 \times S^5$ background ($\tilde{\gamma} = 0$). For simplicity we start with D1-brane that rotates on S^2. We closely follow recent interesting paper [27].

3.1 Single D1-brane on S^2

This case is characterised by condition

$$\tilde{\phi}_2' = 0\,, \quad \omega_2 = 0\ . \tag{3.1}$$

Then the equation of motion for $\tilde{\phi}'_1$ (2.12) implies following relation

$$R^2\kappa^2 = \frac{B\omega_1}{A} \ . \tag{3.2}$$

Since the equation (2.12) does not determine $\tilde{\phi}_1$ we can presume that

$$\tilde{\phi}'_1 = 1 \tag{3.3}$$

so that (A.12) reduces into

$$\det \mathbf{A} = -\alpha^2 R^4 \frac{[\kappa^2(\theta'^2 + \sin^2\theta) - \theta'^2 \sin^2\theta\omega_1^2]}{1 + e^{2\Phi_0}\Pi^2} \ . \tag{3.4}$$

This result together with (2.11) implies following differential equation for θ

$$\theta' = \frac{\sin\theta}{|A|}\left[\frac{e^{-2\Phi_0}\alpha^2\omega_1^2(1 + e^{2\Phi_0}\Pi^2)\sin^2\theta - A^2\kappa^2}{\kappa^2 - \omega_1^2\sin^2\theta}\right]^{1/2} \ . \tag{3.5}$$

This equation is generalisation of the equation derived in paper [27]. In fact, due to the fact that the dilaton is constant and C_2 vanishes for $\gamma = 0$ the dynamics of D1-brane has similar form as the dynamics of fundamental string. More precisely, it is well known that Π determines the number of fundamental strings on the world-volume of D1-brane. Then the equation above determines the dynamics of the bound state of single D1-brane and Π fundamental strings in $AdS_5 \times S^5$ background.

To proceed further we have to impose the boundary condition on the world-volume fields since we consider closed D1-brane. The natural boundary conditions take the form

$$2\pi = \int_0^{2\pi} d\sigma = 2n\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{\alpha|\theta'|} \ , \tag{3.6}$$

where n denotes number of spikes on D1-brane world-volume and where $\theta_{\min}$ and $\theta_{\max}$ will be defined below.

Let us now evaluate charges given in (2.15). Using (2.11) we obtain that P_t is equal to

$$P_t = -\frac{2n\tau_1\kappa R^2}{\alpha\omega_1}\int_{\theta_{\min}}^{\theta_{\max}} d\theta \frac{\sin\theta(e^{-2\Phi_0}\alpha^2\omega_1^2(1 + e^{2\Phi_0}\Pi^2) - A^2\omega_1^2)}{\left[(e^{-2\Phi_0}\alpha^2\omega_1^2(1+e^{2\Phi_0}\Pi^2)\sin^2\theta - A^2\kappa^2)(\kappa^2 - \omega_1^2\sin^2\theta)\right]^{1/2}}. \tag{3.7}$$

In the same way we obtain

$$P_1 = \frac{2n\tau_1 R^2}{\alpha}\int_{\theta_{\min}}^{\theta_{\max}} d\theta \sin\theta\sqrt{\frac{e^{-2\Phi_0}\alpha^2\omega_1^2(1 + e^{2\Phi_0}\Pi^2)\sin^2\theta - A^2\kappa^2}{\kappa^2 - \omega_1^2\sin^2\theta}} \ . \tag{3.8}$$

Finally, the difference between two spikes is given by an expression

$$\triangle\psi = 2\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{\alpha|\theta'|} = -2\int_{\theta_{\min}}^{\theta_{\max}} d\theta \frac{A}{\alpha\sin\theta}\sqrt{\frac{\kappa^2 - \omega_1^2\sin^2\theta}{e^{-2\Phi_0}\alpha^2\omega_1^2(1 + e^{2\Phi_0}\Pi^2)\sin^2\theta - A^2\kappa^2}} \ . \tag{3.9}$$

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Note that this is positive since we have $A < 0$. Now requiring that the arguments in θ' is positive we find the range of θ can be

$$\text{CASE I}: \quad \frac{A^2\kappa^2}{\omega_1^2 e^{-2\Phi_0}\alpha^2(1+e^{2\Phi_0}\Pi^2)} < \sin^2\theta < \frac{\kappa^2}{\omega_1^2} \tag{3.10}$$

or

$$\text{CASE II}: \quad \frac{\kappa^2}{\omega_1^2} < \sin^2\theta < \frac{A^2\kappa^2}{\omega_1^2 e^{-2\Phi_0}\alpha^2(1+e^{2\Phi_0}\Pi^2)} \ . \tag{3.11}$$

Further, in the first case we can have (i)$\frac{\kappa^2}{\omega_1^2} < 1$ or (ii)$\frac{\kappa^2}{\omega_1^2} > 1$. In the second case we have (iii) $\frac{A^2\kappa^2}{\omega_1^2 e^{-2\Phi_0}\alpha^2(1+e^{2\Phi_0}\Pi^2)} < 1$ or (iv)$\frac{A^2\kappa^2}{\omega_1^2 e^{-2\Phi_0}\alpha^2(1+e^{2\Phi_0}\Pi^2)} > 1$. Note that these results can be considered as generalisation of results derived in [27].

3.1.1 First limiting case: giant magnon

Let us consider the case (i) and (ii) given above and take the limit $|\omega_1| \rightarrow \kappa$. Following [27] we define two angles

$$\sin^2\theta_{\min} = \left(\frac{A^2\kappa^2}{\omega_1^2 e^{-2\Phi_0}\alpha^2(1+e^{2\Phi_0}\Pi^2)}\right) , \quad \theta_{\max} = \arcsin\frac{\kappa}{\omega_1} \ , \ \theta_{\min} \leq \theta \leq \theta_{\max} \ . \tag{3.12}$$

The limit $|\omega_1| \rightarrow \kappa$ corresponds to $\theta_{\max} \rightarrow \frac{\pi}{2}$. In this case the equation of motion for θ (3.5) implies

$$\int \frac{d\theta \sin\theta_{\min}\cos\theta}{\sin\theta\sqrt{\sin^2\theta - \sin^2\theta_{\min}}} = \pm d\sigma \tag{3.13}$$

that has solution

$$\sin\theta = \mp\frac{\sin\theta_{\min}}{\sin\sigma} \ . \tag{3.14}$$

Further, $\triangle\psi$ is equal to

$$\triangle\psi = 2\sin\theta_{\min}\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta\cos\theta}{\alpha\sin\theta\sqrt{\sin^2\theta-\sin^2\theta_{\min}}} = \frac{2}{\alpha}\text{arcos}(\sin\theta_{\min}) \tag{3.15}$$

and the energy E is equal to

$$E = -P_t = -\frac{2n\tau_1 R^2 A\kappa^2}{\alpha\omega_1^2}\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta\sin\theta(1-\sin^2\theta_{\min})}{\sin\theta_{\min}\sqrt{(\sin^2\theta-\sin^2\theta_{\min})(\sin^2\theta_{\max}-\sin^2\theta)}} \ . \tag{3.16}$$

In the same way we obtain

$$P_1 = -\frac{2n\tau_1 R^2 A\kappa\omega_1}{\alpha\omega_1^2}\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta\sin\theta}{\sin\theta_{\min}}\frac{\sqrt{\sin^2\theta-\sin^2\theta_{\min}}}{\sqrt{\sin^2\theta_{\max}-\sin^2\theta}} \tag{3.17}$$

and hence

$$E - P_1 = 2ne^{-\Phi_0}\tau_1\sqrt{1+e^{2\Phi_0}\Pi^2}\sin\frac{\triangle\psi}{2} \ . \tag{3.18}$$

We have derived an analogue of the giant magnon dispersion relation for D1-brane with world-volume electric flux Π. Then we can interpret the solution above as the giant magnon on the world-volume of the bound state of single D1-brane and $|\Pi|$ fundamental strings.

3.1.2 Second limiting case: Spike solution

The spike configuration corresponds to the limit

$$\omega^2 \to \frac{e^{-2\Phi_0}\alpha^2(1+e^{2\Phi_0}\Pi^2)}{A^2\kappa^2} \ . \tag{3.19}$$

We again define

$$\sin\theta_{\rm min} = \frac{\kappa}{\omega_1} \ , \quad \sin^2\theta_{\rm max} = \frac{A^2\kappa^2}{e^{-2\Phi_0}\omega_1^2\alpha^2(1+e^{2\Phi_0}\Pi^2)} \tag{3.20}$$

so that the limit (3.19) corresponds to $\theta_{\rm max} \to \frac{\pi}{2}$. Then the differential equation for θ' (3.5) takes the form

$$\theta' = \frac{\sin\theta_{\rm min}\sin\theta}{\sin\theta_{\rm max}}\sqrt{\frac{\sin^2\theta_{\rm max}-\sin^2\theta}{\sin^2\theta-\sin^2\theta_{\rm min}}} \tag{3.21}$$

that can be easily integrated with the result

$$\frac{\cos\theta_{\rm min}}{\sin\theta_{\rm min}}\cosh^{-1}\left(\frac{\cos\theta_{\rm min}}{\cos\sigma}\right) - \cos^{-1}\left(\frac{\sin\theta_{\rm min}}{\sin\theta}\right) = \pm\sigma \ . \tag{3.22}$$

Further, for the limit (3.19) the charge P_1 given in (3.8) is equal to

$$P_1 = 2n\tau_1 R^2 e^{-\Phi_0}\sqrt{1+e^{2\Phi_0}\Pi^2}\cos\theta_{\rm min} \ . \tag{3.23}$$

On the other hand we obtain that E and $\triangle\psi$ derived in (3.7) and (3.9) diverge. However we can find combinations of these charges that is finite

$$E - ne^{-\Phi_0}\sqrt{1+e^{2\Phi_0}\Pi^2}\tau_1 R^2\alpha\triangle\psi = 2n\tau_1 R^2 e^{-\Phi_0}\sqrt{1+e^{2\Phi_0}\Pi^2}(\frac{\pi}{2}-\theta_{\rm min}) \ . \tag{3.24}$$

Again, this result can be considered as a generalisation of the spike relation derived in [27] to the case of bound state of single D1-brane and collection of Π fundamental strings.

3.2 D1-brane on S^3: two angular momenta

Now consider more general situation when we examine the motion of D1-brane with an extra angular momentum. In this case the equations of motion for $\tilde\phi_1$ (2.12) and for $\tilde\phi_2$ (2.13) are equal to

$$\begin{aligned} &- AR^2\sin^2\theta\frac{[\omega_2\cos^2\theta(\tilde\phi_1'\omega_2-\tilde\phi_2'\omega_1)-\phi_1'\kappa^2]}{\sin^2\theta\omega_1\tilde\phi_1'+\cos^2\theta\omega_2\tilde\phi_2'} = B \ , \\ &- AR^2\cos^2\theta\frac{[\omega_1\sin^2\theta(\tilde\phi_2'\omega_1-\tilde\phi_1'\omega_2)-\tilde\phi_2'\kappa^2]}{\sin^2\theta\omega_1\tilde\phi_1'+\cos^2\theta\omega_2\tilde\phi_2'} = C \ . \end{aligned} \tag{3.25}$$

It turns out that if we combine these two equations we obtain relations between constants A, B and C. In other words we are free to presume particular form of either $\tilde\phi_1$ or $\tilde\phi_2$ and we choose the simplest one

$$\tilde\phi_1' = 1 \ . \tag{3.26}$$

Then with the help of the equation of motion for ϕ_1 we find

$$\tilde{\phi}_2' = \frac{\sin^2\theta(AR^2\kappa^2 - B\omega_1 - AR^2\omega_2^2\cos^2\theta)}{\omega_2\cos^2\theta(B - AR^2\omega_1\sin^2\theta)} \ . \tag{3.27}$$

Following [27] we choose the constants of motion appropriately so that

$$\theta' \to 0 \text{ as } \theta \to \frac{\pi}{2} \ . \tag{3.28}$$

It turns out that the natural choice is

$$A = \frac{e^{-\Phi_0}\alpha\omega_1}{\kappa}\sqrt{1 + e^{2\Phi_0}\Pi^2}\,, \quad B = \alpha e^{-\Phi_0}R^2\kappa\sqrt{1 + e^{2\Phi_0}\Pi^2} \ . \tag{3.29}$$

Then the equation of motion (3.27) simplifies considerably

$$\tilde{\phi}_2' = \frac{\sin^2\theta\omega_1\omega_2}{\omega_1^2\sin^2\theta - \kappa^2} \ . \tag{3.30}$$

Further, using (2.11) and (3.30) we easily find differential equation for θ'

$$\theta' = \frac{|\kappa|\sin\theta\cos\theta}{\omega_1^2\sin^2\theta - \kappa^2}\sqrt{(\omega_1^2 - \omega_2^2)\sin^2\theta - \kappa^2} \tag{3.31}$$

and consequently

$$\begin{aligned}
P_t &= -2n\tau_1 e^{-\Phi_0}R^2\sqrt{1 + e^{2\Phi_0}\Pi^2}\int_{\theta_{\min}}^{\theta_{\max}} d\theta\frac{\sin\theta(\kappa^2 - \omega_1^2)}{\kappa\cos\theta\sqrt{(\omega_1^2 - \omega_2^2)\sin^2\theta - \kappa^2}}\,, \\
P_1 &= 2n\tau_1 e^{-\Phi_0}R^2\sqrt{1 + e^{2\Phi_0}\Pi^2}\int_{\theta_{\min}}^{\theta_{\max}} d\theta\frac{\omega_1\sin\theta\cos\theta}{\sqrt{(\omega_1^2 - \omega_2^2)\sin^2\theta - \kappa^2}}\,, \\
P_2 &= -2n\tau_1 e^{-\Phi_0}R^2\sqrt{1 + e^{2\Phi_0}\Pi^2}\int_{\theta_{\min}}^{\theta_{\max}} d\theta\frac{\omega_2\sin\theta\cos\theta}{\sqrt{(\omega_1^2 - \omega_2^2)\sin^2\theta - \kappa^2}} \ .
\end{aligned} \tag{3.32}$$

Finally we find that the difference in angle between two endpoints of the string is equal to

$$\triangle\psi = -2\int_{\theta_{\min}}^{\theta_{\max}} d\theta\frac{\omega_1^2\sin^2\theta - \kappa^2}{\alpha\kappa\sin\theta\cos\theta\sqrt{(\omega_1^2 - \omega_2^2)\sin^2\theta - \kappa^2}} \ . \tag{3.33}$$

In all these calculation $\theta_{\max} = \frac{\pi}{2}$ and $\theta_{\min} = \arcsin\left(\frac{|\kappa|}{\sqrt{\omega_1^2 - \omega_2^2}}\right)$ where we presume $\omega_1^2 > \omega_2^2$. Here we have chosen $\theta_{\min}$ such that insider square root is positive. Then, since $\arcsin\left(\frac{|\kappa|}{\omega_1}\right) < \arcsin\left(\frac{|\kappa|}{\sqrt{\omega_1^2 - \omega_2^2}}\right) < \pi/2$, θ can never reach a value such that $\sin\theta = \frac{|\kappa|}{\omega_1}$. Thus in this case θ' cannot go to infinity at any point. Performing integrals we obtain

$$\begin{aligned}
P_1 &= 2n\tau_1 e^{-\Phi_0}R^2\sqrt{1 + e^{2\Phi_0}\Pi^2}\frac{1}{\cos\gamma}\sin\overline{\theta}\,, \\
P_2 &= -2n\tau_1 e^{-\Phi_0}R^2\sqrt{1 + e^{2\Phi_0}\Pi^2}\frac{\sin\gamma}{\cos\gamma}\sin\overline{\theta}\,,
\end{aligned} \tag{3.34}$$

where we have

$$\sin\overline{\theta} = \frac{\sqrt{\omega_1^2 - \omega_2^2 - \kappa^2}}{\sqrt{\omega_1^2 - \omega_2^2}} , \quad \sin\gamma = \frac{\omega_2}{\omega_1} . \tag{3.35}$$

Finally we find

$$E - n\tau_1 R^2 e^{-\Phi_0}\sqrt{1 + e^{2\Phi_0}\Pi^2}\alpha\triangle\psi = 2n\tau_1 R^2 e^{-\Phi_0}\sqrt{1 + e^{2\Phi_0}\Pi^2}\overline{\theta} , \tag{3.36}$$

where

$$\overline{\theta} = \frac{\pi}{2} - \theta_0 , \quad \sin\theta_0 = \frac{|\kappa|}{\sqrt{\omega_1^2 - \omega_2^2}} . \tag{3.37}$$

Then we can also write

$$P_1 = \sqrt{P_2^2 + 2n\tau_1 R^2 e^{-\Phi_0}\sqrt{1 + e^{2\Phi_0}\Pi^2}\sin^2\overline{\theta}} . \tag{3.38}$$

Again this result can be thought of as a generalisation of the results presented in [27] to the case of bound state of single D1-brane and Π fundamental strings.

In the rest of the paper we will study the dynamics of D1-brane in β-deformed background. Before we come to this problem we review some properties of Nambu-Goto form of the string action in this background in appendix-B.

4. D1-brane in β-deformed background

In this section we return to the study of non-trivial solutions on the world-volume of D1-brane in β-deformed background. We closely follow the study of the fundamental string performed in previous section. Recall that the equation of motion for A_α implies an existence of conserved quantity Π defined in (A.9) that has the physical meaning as the number of fundamental strings. In analogy with the discussion performed in previous section we fix the diffeomorphism invariance by imposing the conditions

$$\sqrt{-\det\mathbf{A}}\sqrt{1 + e^{2\Phi_0}\Pi^2 G} = \mathbf{A}_{\sigma\sigma} , \quad \mathbf{A}_{\tau\tau} = 0 \tag{4.1}$$

or equivalently

$$(\mathbf{A}_{\tau\sigma})^S = \mathbf{A}_{\sigma\sigma} . \tag{4.2}$$

For this ansatz the conserved charges P_t, J_1 and J_2 and the equations of motion for ϕ_1 and ϕ_2 are summarized in appendix-C.

Using the condition $\mathbf{A}_{\tau\tau} = 0$ together with the equations of motion for ϕ_1, ϕ_2 in (C.2), (C.3) respectively, and also the relation (C.4) we obtain differential equation for θ'^2 in the form

$$\begin{aligned}\theta'^2 = {} & \kappa^2\frac{\beta^2 + 2\alpha^2 - 2\alpha\beta}{(2\alpha\beta - \beta)^2} - \\ & - \frac{B^2 e^{2\Phi_0}}{R^4\sin^2\theta(1 + e^{2\Phi_0}\Pi^2 G)(2\alpha\beta - \beta^2)^2} - \frac{C^2 e^{2\Phi_0}}{R^4\cos^2\theta(1 + e^{2\Phi_0}\Pi^2 G)(2\alpha\beta - \beta^2)^2} - \\ & - \frac{2\tilde{\gamma}\alpha G e^{2\Phi_0}\Pi}{(1 + e^{2\Phi_0}\Pi^2 G)R^2(2\alpha\beta - \beta^2)^2}(\omega_1 C\sin^2\theta - \omega_2 B\cos^2\theta) - \\ & - \frac{\alpha^2 G(\omega_1^2\sin^2\theta + \omega_2^2\cos^2\theta)}{(2\alpha\beta - \beta^2)^2}\left(\frac{1 + e^{2\Phi_0}\Pi^2}{1 + e^{2\Phi_0}\Pi^2 G}\right) .\end{aligned} \tag{4.3}$$

We must however stress one important point. The equations of motion given above are valid in case when $\Pi \gg 1$. This follows from the analysis of the equation of motion for $x^0 = t$

$$\kappa R^2 \left[\alpha \frac{e^{-\Phi}\mathbf{A}_{\sigma\sigma}}{\sqrt{-\det \mathbf{A}}} - \alpha \frac{e^{-\Phi}(\mathbf{A}_{\tau\sigma})^S}{\sqrt{-\det \mathbf{A}}}\right]' = \kappa R^2[(\alpha-\beta)e^{-\Phi}\sqrt{1+e^{2\Phi}\Pi^2}]' = 0 \tag{4.4}$$

and we see that this equation is obeyed for general Π in case when

$$\alpha = \beta \ . \tag{4.5}$$

On the other hand for $\Pi \gg 1$ we can write $1 + Ge^{2\Phi_0}\Pi^2 \approx Ge^{2\Phi_0}\Pi^2$ and we see that (4.4) is automatically obeyed. Let us now analyse this situation in more detail.

4.1 $\Pi \gg 1$

This situation corresponds to the bound state of large number of fundamental strings Π and one $D1$-brane. In this case it is natural to perform a rescaling $B = b\Pi\,, C = c\Pi$. Then in the limit $\Pi \gg 1$ the equations (C.2), (C.3) take the form

$$\begin{aligned}
\tilde{\phi}_1' &= \frac{1}{2\alpha\beta-\beta^2}\left[\frac{b}{R^2 G\sin^2\theta} - \alpha\omega_2\tilde{\gamma}\cos^2\theta + (\beta\omega_1 - \alpha\omega_1)\right] , \\
\tilde{\phi}_2' &= \frac{1}{2\alpha\beta-\beta^2}\left[\frac{c}{R^2 G\cos^2\theta} + \alpha\omega_1\tilde{\gamma}\sin^2\theta + (\beta\omega_2 - \alpha\omega_2)\right] .
\end{aligned} \tag{4.6}$$

Finally, the equation (4.3) reduces into

$$\begin{aligned}
\theta'^2 &= \kappa^2\frac{\beta^2+2\alpha^2-2\alpha\beta}{(2\alpha\beta-\beta)^2} - \frac{\alpha^2(\omega_1^2\sin^2\theta+\omega_2^2\cos^2\theta)}{(2\alpha\beta-\beta^2)^2} - \\
&-\frac{b^2}{R^4G\sin^2\theta(2\alpha\beta-\beta^2)^2} - \frac{c^2}{R^4\cos^2\theta G(2\alpha\beta-\beta^2)^2} - \\
&-\frac{2\tilde{\gamma}\alpha}{R^2(2\alpha\beta-\beta^2)^2}(\omega_1 c\sin^2\theta - \omega_2 b\cos^2\theta) \ .
\end{aligned} \tag{4.7}$$

We see that the equations (4.6) and (4.7) take exactly the same form as the equations (B.15), (B.17) and (B.18)[5] that describe the dynamics of fundamental string. Further, in the limit $e^{\Phi_0}\Pi \gg 1$ charges (C.1) reduce into

$$\begin{aligned}
P_t &= -\tau_1 R^2|\Pi|\kappa\int_0^{2\pi} d\sigma \ , \\
J_1 &= \tau_1 R^2|\Pi|\int_0^{2\pi} d\sigma G\sin^2\theta[\omega_1 + (\beta-\alpha)\tilde{\phi}_1' - \tilde{\gamma}\alpha\cos^2\theta\tilde{\phi}_2'] \ , \\
J_2 &= \tau_1 R^2|\Pi|\int_0^{2\pi} d\sigma G\cos^2\theta[\omega_2 + (\beta-\alpha)\tilde{\phi}_2' + \tilde{\gamma}\alpha\sin^2\theta\tilde{\phi}_1'] \ .
\end{aligned} \tag{4.8}$$

that exactly reproduce the form of these charges for collection of fundamental strings.

[5]After appropriate identification of parameters α and β and constants b, c.

4.2 General Π

As we have seen above the only solution of the equation of motion for x^0 corresponds to $\alpha = \beta$. In this case the equation of motion for $\tilde{\phi}_1, \tilde{\phi}_2$ take the form

$$\begin{aligned}
\tilde{\phi}_1' &= \frac{1}{\sqrt{1+e^{2\Phi_0}G\Pi^2\alpha^2}}\left[\frac{Be^{\Phi_0}}{R^2\sqrt{G}\sin^2\theta} - \alpha\omega_2\Pi e^{\Phi_0}\tilde{\gamma}\sqrt{G}\cos^2\theta\right] . \\
\tilde{\phi}_2' &= \frac{1}{\sqrt{1+e^{2\Phi_0}G\Pi^2\alpha^2}}\left[\frac{Ce^{\Phi_0}}{R^2\sqrt{G}\cos^2\theta} + \alpha\omega_1\Pi e^{\Phi_0}\tilde{\gamma}\sqrt{G}\sin^2\theta\right] .
\end{aligned} \tag{4.9}$$

Further, the condition (4.2) for $\beta = \alpha$ implies

$$B\omega_1 + C\omega_2 = 0 . \tag{4.10}$$

Then, using the condition $\mathbf{A}_{\tau\tau} = 0$ and (4.10) we finally obtain differential equation for θ

$$\begin{aligned}
\theta'^2 = \frac{\kappa^2}{\alpha^2} + (\omega_1^2\sin^2\theta + \omega_2^2\cos^2\theta)\Bigg[&\frac{2\tilde{\gamma}Ge^{2\Phi_0}\Pi B}{\omega_2(1+e^{2\Phi_0}\Pi^2G)R^2\alpha^3} - \\
&- \frac{B^2e^{2\Phi_0}}{R^4\alpha^4\omega_2^2\sin^2\theta\cos^2\theta(1+e^{2\Phi_0}\Pi^2G)} - \frac{G}{\alpha^2}\left(\frac{1+e^{2\Phi_0}\Pi^2}{1+e^{2\Phi_0}\Pi^2G}\right)\Bigg] .
\end{aligned} \tag{4.11}$$

Note also that for $\alpha = \beta$ the charges (C.1) take the form

$$\begin{aligned}
P_t &= -\tau_1R^2\kappa\int_0^{2\pi} d\sigma\frac{e^{-\Phi_0}}{\sqrt{G}}\sqrt{1+e^{2\Phi_0}G\Pi^2} , \\
J_1 &= \tau_1R^2\int_0^{2\pi} d\sigma[e^{-\Phi_0}\omega_1\sqrt{G}\sin^2\theta\sqrt{1+e^{2\Phi_0}G\Pi^2} - \tilde{\gamma}G\cos^2\theta\sin^2\theta\alpha\tilde{\phi}_2'\Pi] , \\
J_2 &= \tau_1R^2\int_0^{2\pi} d\sigma[e^{-\Phi_0}\omega_2\sqrt{G}\cos^2\theta\sqrt{1+e^{2\Phi_0}G\Pi^2} + \tilde{\gamma}G\cos^2\theta\sin^2\theta\alpha\tilde{\phi}_1'\Pi] .
\end{aligned} \tag{4.12}$$

It is still difficult to solve the equations (4.9) and (4.11) for any value of Π. In fact we were not able to find time-dependent configuration that has an interpretation as giant magnon. For that reason we now restrict to the case of constant θ_c. First of all we obtain that ϕ_1 and ϕ_2 have following solutions

$$\begin{aligned}
\phi_1 &= \omega_1\tau + \tilde{\phi}_1'(\theta_c)(\alpha\sigma + \beta\tau) , \\
\phi_2 &= \omega_2\tau + \tilde{\phi}_2'(\theta_c)(\alpha\sigma + \beta\tau) ,
\end{aligned} \tag{4.13}$$

where $\tilde{\phi}_{1,2}'(\theta_c)$ are constants whose explicit values are given in (4.9) evaluated for θ_c. Note that the periodicity conditions for ϕ_1, ϕ_2 imply

$$\begin{aligned}
\phi_1(2\pi) - \phi_1(0) &= \tilde{\phi}_1'(\theta_c)\alpha 2\pi = n_1 2\pi , \\
\phi_2(2\pi) - \phi_2(0) &= \tilde{\phi}_2'(\theta_c)\alpha 2\pi = n_2 2\pi ,
\end{aligned} \tag{4.14}$$

where $n_{1,2}$ are winding numbers. Further, the equation $\mathbf{A}_{\tau\tau} = 0$ implies the relation between $\kappa, \omega_{1,2}, n_{1,2}$ and θ_c in the form

$$0 = \kappa^2(1+\tilde{\gamma}^2\sin^2\theta_c\cos^2\theta_c) - \sin^2\theta_c(\omega_1+n_1)^2 - \cos^2\theta_c(\omega_2+n_2)^2 = 0 . \tag{4.15}$$

We also have to demand that θ_c solves the equation of motion for θ. In fact, after some algebra we obtain following equation

$$\tilde{\gamma}^2(\cos^2\theta_c - \sin^2\theta_c)G\left[\sin^2\theta_c\omega_1^2 + \cos^2\theta_c\omega_2^2 + \right.$$
$$\left. + G\sin^2\theta_c(\omega_1^2+n_1^2)e^{2\Phi_0}\Pi^2 + G\cos^2\theta_c(\omega_2^2+n_2^2)e^{2\Phi_0}\Pi^2\right] -$$
$$-(\omega_1^2 - n_1^2 - \omega_2^2 + n_2^2)\left(1 + Ge^{2\Phi_0}\Pi^2\right) = 0 \ . \tag{4.16}$$

Finally, if we impose the condition $\mathbf{A}_{\sigma\sigma} = (\mathbf{A}_{\tau\sigma})^S$ we obtain

$$\sin^2\theta_c n_1\omega_1 + \cos^2\theta_c n_2\omega_2 = 0 \ . \tag{4.17}$$

We solve this equation with the ansatz

$$\omega_1 = \omega_2 \equiv \omega \ , \quad n_1 = -n_2 \equiv n \ , \quad \theta_c = \frac{\pi}{4} \ . \tag{4.18}$$

Then it is easy to see that (4.16) is obeyed for the ansatz (4.18). In what follows we restrict ourselves to this particular situation. Then for the ansatz (4.18) the equation (4.15) implies

$$\kappa^2 = \frac{1}{1+\frac{\tilde{\gamma}^2}{4}}(\omega^2+n^2) \ . \tag{4.19}$$

Note also that for the ansatz (4.18) the charges (4.12) take the form

$$\begin{aligned} P_t &= -\tau_1 R^2 e^{-\Phi_0} 2\pi\kappa\sqrt{1+\frac{\tilde{\gamma}^2}{4}+e^{2\Phi_0}\Pi^2} \ , \\ J_1 &= J_2 \equiv \frac{1}{2}J = \tau_1 R^2 e^{-\Phi_0} 2\pi \frac{1}{2(1+\frac{\tilde{\gamma}^2}{4})}\left[\omega\sqrt{1+\frac{\tilde{\gamma}^2}{4}+e^{2\Phi_0}\Pi^2} + \frac{n}{2}\tilde{\gamma}e^{\Phi_0}\Pi\right] \ . \end{aligned} \tag{4.20}$$

Finally, using (4.19) we find following relation between $E = -P_t$ and J, Π and n

$$E^2 = J^2 + \left(-2\pi n\tau_1 R^2\Pi + \frac{\tilde{\gamma}}{2}J\right)^2 + (2\pi R^2\tau_1 e^{-\Phi_0}n)^2 \ . \tag{4.21}$$

First two terms above exactly reproduce the results derived in paper [46][6] where the term proportional to Π describes contribution from wrapping fundamental string. The last term in (4.21) follows from the contribution of wrapped D1-brane.

5. Conclusions

This paper has been devoted to the study of dynamics of D1-brane in the $AdS_5 \times S^5$ background and also in its β-deformed version. We wanted to see how D1-brane dynamics is different from the corresponding study of the fundamental string. In case of $AdS_5 \times S^5$ we have derived the straightforward generalisation of the giant magnon and spike solutions that were found in case of fundamental strings [7, 27]. More precisely, we have found giant magnon and spike configurations that are related to the dynamics of bound state of single

[6]The minus sign in front of n is irrelevant.

D1-brane and Π fundamental strings. We mean that this is very satisfactory result that explicitly demonstrates similarity of the classical description of fundamental string and D1-brane in $AdS_5 \times S^5$ background.

Then we proceed to the analysis of D1-brane in β-deformed background. Now we have found that the situation is different. In fact, we showed that in case of the large number of fundamental strings that are stretched along world-volume of D1-brane the dynamics of this system takes the same form as in case of the fundamental string [33]. This result again demonstrates the consistency of our approach. On the other hand in case of finite number of fundamental strings we were not able to find time dependent configurations that could be interpreted as giant spikes or magnons on the world-volume of D1-brane. We mean that this is a consequence of the fact that classical D1-brane explicitly couples to dilaton which is non-trivial in the β-deformed background and has significant contribution to the dynamics of D1-brane. On the other hand when we have restricted to the study of dynamics of D1-brane with constant θ we have been able to find the generalisation of the formula derived in [46].

In summary, we hope that our result could be useful for further study of the dynamics of D1-brane in $AdS_5 \times S^5$ background and its deformation. It would be certainly interesting to study properties of D1-brane in the β-deformed $AdS_5 \times S^5$ background with complex deformation parameter.

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A. Equations of motion

Varying the action (2.1) with respect to x^M we obtain the following equations of motion for x^M[7]

$$\begin{aligned} &- \tau_1 \partial_M [e^{-\Phi}] \sqrt{-\det \mathbf{A}} - \\ -\frac{\tau_1}{2} e^{-\Phi} (\partial_M g_{KL} \partial_\alpha x^K \partial_\beta x^L - \partial_M b_{KL} \partial_\alpha x^K \partial_\beta x^L) &\left(\mathbf{A}^{-1}\right)^{\beta\alpha} \sqrt{-\det \mathbf{A}} + \\ +\tau_1 \partial_\alpha [e^{-\Phi} g_{MN} \partial_\beta x^N &\left(\mathbf{A}^{-1}\right)^{\beta\alpha}_S \sqrt{-\det \mathbf{A}}] - \\ -\tau_1 \partial_\alpha [e^{-\Phi} b_{MN} \partial_\beta x^N &\left(\mathbf{A}^{-1}\right)^{\beta\alpha}_A \sqrt{-\det \mathbf{A}}] + J_M = 0 \, , \end{aligned} \quad (A.1)$$

where

$$J_M = \frac{\delta S_{WZ}}{\delta x^M} \quad (A.2)$$

[7] equations of motion for all the branes in AdS spacetime has been discussed in [47]

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and where

$$\left(\mathbf{A}^{-1}\right)_S^{\alpha\beta} = \frac{1}{2}\left(\left(\mathbf{A}^{-1}\right)^{\alpha\beta} + \left(\mathbf{A}^{-1}\right)^{\beta\alpha}\right) , \quad \left(\mathbf{A}^{-1}\right)_A^{\alpha\beta} = \frac{1}{2}\left(\left(\mathbf{A}^{-1}\right)^{\alpha\beta} - \left(\mathbf{A}^{-1}\right)^{\beta\alpha}\right) . \quad \text{(A.3)}$$

In the same way the variation of (2.1) with respect to A_α implies following equation of motion

$$2\pi\alpha'\tau_1\partial_\beta[e^{-\Phi}\left(\mathbf{A}^{-1}\right)_A^{\beta\alpha}\sqrt{-\det\mathbf{A}}] + J^\alpha = 0 , \quad \text{(A.4)}$$

where

$$J^\alpha = \frac{\delta S_{WZ}}{\delta A_\alpha} . \quad \text{(A.5)}$$

Now we start to analyse the equations of motions given in (A.1) and (A.4). To begin with note that for the ansatz (2.9) the matrix $\mathbf{A}$ is equal to

$$\begin{aligned}
\mathbf{A}_{\tau\tau} &= R^2[-\kappa^2 + \beta^2\theta'^2 + G\sin^2\theta(\omega_1 + \beta\tilde{\phi}_1')^2 + G\cos^2\theta(\omega_2 + \beta\tilde{\phi}_2')^2] , \\
\mathbf{A}_{\tau\sigma} &= R^2[\alpha\beta\theta'^2 + G\sin^2\theta\alpha(\omega_1 + \beta\tilde{\phi}_1')\tilde{\phi}_1' + G\cos^2\theta\alpha(\omega_2 + \beta\tilde{\phi}_2')\tilde{\phi}_2' + \\
&\quad + \tilde{\gamma}G\sin^2\theta\cos^2\theta\alpha(\omega_2\tilde{\phi}_1' - \omega_1\tilde{\phi}_2')] + 2\pi\alpha' F , \\
\mathbf{A}_{\sigma\tau} &= R^2[\alpha\beta\theta'^2 + G\sin^2\theta\alpha(\omega_1 + \beta\tilde{\phi}_1')\tilde{\phi}_1' + G\cos^2\theta\alpha(\omega_2 + \beta\tilde{\phi}_2')\tilde{\phi}_2' - \\
&\quad - \tilde{\gamma}G\sin^2\theta\cos^2\theta\alpha(\omega_2\tilde{\phi}_1' - \omega_1\tilde{\phi}_2')] - 2\pi\alpha' F , \\
\mathbf{A}_{\sigma\sigma} &= R^2[\alpha^2\theta'^2 + G\sin^2\theta\alpha^2\tilde{\phi}_1'^2 + G\cos^2\theta\alpha^2\tilde{\phi}_2'^2] ,
\end{aligned} \quad \text{(A.6)}$$

where $F_{\tau\sigma} \equiv F$ and where $(\ldots)' \equiv \frac{d}{dy}(\ldots)$. Then it is easy to calculate $\det\mathbf{A}$ and we obtain

$$\begin{aligned}
\det\mathbf{A} &= -\alpha^2R^4\kappa^2[\theta'^2 + G\sin^2\theta\tilde{\phi}_1'^2 + G\cos^2\theta\tilde{\phi}_2'^2] + \\
&\quad + \alpha^2R^4G^2\cos^2\theta\sin^2\theta(\omega_1\tilde{\phi}_2' - \omega_2\tilde{\phi}_1')^2 + \\
&\quad + \alpha^2R^4G\theta'^2(\sin^2\theta\omega_1^2 + \cos^2\theta\omega_2^2) + \\
&\quad + [\tilde{\gamma}R^2G\sin^2\theta\cos^2\theta\alpha(\omega_2\tilde{\phi}_1' - \omega_1\tilde{\phi}_2') + 2\pi\alpha' F]^2 .
\end{aligned} \quad \text{(A.7)}$$

Let us now return to the equations of motion (A.1) and (A.4). The equation of motion for A_α implies

$$\begin{aligned}
\partial_\tau[e^{-\Phi}\left(\mathbf{A}^{-1}\right)_A^{\tau\sigma}\sqrt{-\det\mathbf{A}}] &= 0 , \\
\partial_\sigma[e^{-\Phi}\left(\mathbf{A}^{-1}\right)_A^{\sigma\tau}\sqrt{-\det\mathbf{A}}] &= 0
\end{aligned} \quad \text{(A.8)}$$

and consequently

$$e^{-\Phi}\frac{(\mathbf{A}_{\tau\sigma})_A}{\sqrt{-\det\mathbf{A}}} = \Pi , \quad \text{(A.9)}$$

where Π is constant that counts the number of fundamental strings stretched along world-volume of D1-brane. Further, using the properties of matrices $\mathbf{A}^S$ and $\mathbf{A}^A$ defined in (A.3) we easily find

$$\begin{aligned}
\det\mathbf{A} &= \mathbf{A}_{\tau\tau}\mathbf{A}_{\sigma\sigma} - \mathbf{A}_{\tau\sigma}\mathbf{A}_{\sigma\tau} = \\
&\quad \mathbf{A}_{\tau\tau}\mathbf{A}_{\sigma\sigma} - (\mathbf{A}_{\tau\sigma})^S(\mathbf{A}_{\tau\sigma})^S + (\mathbf{A}_{\tau\sigma})^A(\mathbf{A}_{\tau\sigma})^A .
\end{aligned} \quad \text{(A.10)}$$

If we combine (A.9) with (A.10) we can express $(\mathbf{A}_{\tau\sigma})^A$ as

$$(\mathbf{A}_{\tau\sigma})^A(\mathbf{A}_{\tau\sigma})^A(e^{-2\Phi}+\Pi^2)=(-\mathbf{A}_{\tau\tau}\mathbf{A}_{\sigma\sigma}+(\mathbf{A}_{\tau\sigma})^S(\mathbf{A}_{\tau\sigma})^S)\Pi^2 \tag{A.11}$$

and hence the determinant $\det\mathbf{A}$ takes the form

$$\det\mathbf{A}=\frac{\alpha^2R^4}{1+Ge^{2\Phi_0}\Pi^2}\times\Big[\kappa^2(\theta'^2+G(\sin^2\theta\tilde{\phi}_1'^2+\cos^2\theta\tilde{\phi}_2'^2))- \\ -G^2\cos^2\theta\sin^2\theta(\omega_1\tilde{\phi}_2'-\omega_2\tilde{\phi}_1')^2-\theta'^2G(\sin^2\theta\omega_1^2+\cos^2\theta\omega_2^2)\Big]\ . \tag{A.12}$$

A.1 Conserved charges

The conserved currents are given by

$$\begin{aligned}\mathcal{J}_t^\alpha&=\frac{\delta\mathcal{L}}{\delta\partial_\alpha t}=-\tau_1e^{-\Phi}g_{tt}\partial_\beta t\left(\mathbf{A}^{-1}\right)_S^{\beta\alpha}\sqrt{-\det\mathbf{A}}\,,\\ \mathcal{J}_1^\alpha&=\frac{\delta\mathcal{L}}{\delta\partial_\alpha\phi_1}=-\tau_1e^{-\Phi}[g_{\phi_1\phi_1}\partial_\beta\phi_1\left(\mathbf{A}^{-1}\right)_S^{\beta\alpha}+b_{\phi_1\phi_2}\partial_\beta\phi_2\left(\mathbf{A}^{-1}\right)_A^{\beta\alpha}]\sqrt{-\det\mathbf{A}}\,,\\ \mathcal{J}_2^\alpha&=\frac{\delta\mathcal{L}}{\delta\partial_\alpha\phi_2}=-\tau_1e^{-\Phi}[g_{\phi_2\phi_2}\partial_\beta\phi_2\left(\mathbf{A}^{-1}\right)_S^{\beta\alpha}+b_{\phi_2\phi_1}\partial_\beta\phi_1\left(\mathbf{A}^{-1}\right)_A^{\beta\alpha}]\sqrt{-\det\mathbf{A}}\,.\end{aligned} \tag{A.13}$$

B. Nambu-Goto form of the string action in β-deformed background

In order to understand better the dynamics of D1-brane in β-deformed background we consider Nambu-Goto action for fundamental string in this background. Our goal is to explicitly see how analysis of this action can be related to the analysis of the sigma model form of the action presented in [33].

Let us start with the Nambu-Goto action for fundamental string in general background

$$S=-\frac{1}{2\pi\alpha'}\int d\tau d\sigma\left[\sqrt{-\det\mathbf{a}_{\alpha\beta}}+\frac{1}{2}\varepsilon^{\alpha\beta}b_{MN}\partial_\alpha x^M\partial_\beta x^N\right]\,, \tag{B.1}$$

where $\varepsilon^{\alpha\beta}=-\varepsilon^{\beta\alpha}\,,\quad\varepsilon^{01}=1\,,\quad\mathbf{a}_{\alpha\beta}=g_{MN}\partial_\alpha x^M\partial_\beta x^N$. Variation of (B.1) with respect to x^M implies following equation of motion

$$\frac{1}{2}\partial_Mg_{KL}\partial_\alpha x^K\partial_\beta x^L\left(\mathbf{a}^{-1}\right)^{\beta\alpha}\sqrt{-\det\mathbf{a}}-\partial_\alpha[g_{MN}\partial_\beta x^N\left(\mathbf{a}^{-1}\right)^{\beta\alpha}\sqrt{-\det\mathbf{a}}]+ \\ +\frac{1}{2}\varepsilon^{\alpha\beta}\partial_Mb_{KL}\partial_\alpha x^K\partial_\beta x^L-\partial_\alpha[\varepsilon^{\alpha\beta}b_{MN}\partial_\beta x^N]=0\ . \tag{B.2}$$

For reader's convenience we again write the relevant part of the β-deformed $AdS_5\times S^5$ background

$$ds^2=R^2(-dt^2+d\theta^2+G\sin^2\theta d\phi_1^2+G\cos^2\theta d\phi_2^2) \tag{B.3}$$

and

$$B_{\phi_1\phi_2}=R^2\tilde{\gamma}G\sin^2\theta\cos^2\theta\,,\quad e^{2\Phi}=e^{2\Phi_0}G\,,\quad G=\frac{1}{1+\tilde{\gamma}^2\sin^2\theta\cos^2\theta}\ . \tag{B.4}$$

Let us now consider following ansatz

$$t=\kappa\tau\,,\quad\theta=\theta(y)\,,\quad\phi_1=\omega_1\tau+\tilde{\phi}_1(y)\,,\quad\phi_2=\omega_2\tau+\tilde{\phi}_2(y)\,, \tag{B.5}$$

where

$$y \equiv \alpha\sigma + \beta\tau \ . \tag{B.6}$$

For this ansatz components of matrix $\mathbf{a}$ take the form

$$\begin{aligned}
\mathbf{a}_{\tau\tau} &= R^2[-\kappa^2 + \beta^2\theta'^2 + G\sin^2\theta(\omega_1 + \beta\tilde{\phi}_1')^2 + G\cos^2\theta(\omega_2 + \beta\tilde{\phi}_2')^2] \,, \\
\mathbf{a}_{\tau\sigma} &= \mathbf{a}_{\sigma\tau} = R^2[\alpha\beta\theta'^2 + G\sin^2\theta\alpha(\omega_1 + \beta\tilde{\phi}_1')\tilde{\phi}_1' + G\cos^2\theta\alpha(\omega_2 + \beta\tilde{\phi}')\tilde{\phi}_2'] \,, \\
\mathbf{a}_{\sigma\sigma} &= R^2\alpha^2[\theta'^2 + G\sin^2\theta\tilde{\phi}_1'^2 + G\cos^2\theta\tilde{\phi}_2'^2]
\end{aligned} \tag{B.7}$$

and consequently

$$\begin{aligned}
\det\mathbf{a} = &-\alpha^2R^4\kappa^2[\theta'^2 + G\sin^2\theta\tilde{\phi}_1'^2 + \\
&+G\cos^2\theta\tilde{\phi}_2'^2] + \alpha^2R^4G^2\cos^2\theta\sin^2\theta(\omega_1\tilde{\phi}_2' - \omega_2\tilde{\phi}_1')^2 + \\
&+\theta'^2\alpha^2G(\sin^2\theta\omega_1^2 + \cos^2\theta\omega_2^2) \ .
\end{aligned} \tag{B.8}$$

Note that the fundamental string has following conserved currents

$$\begin{aligned}
\mathcal{J}_t^\alpha &= \frac{\mathcal{L}_{NG}}{\partial_\alpha t} = -\frac{1}{2\pi\alpha'}g_{tt}\partial_\beta t\left(\mathbf{a}^{-1}\right)^{\beta\alpha}\sqrt{-\det\mathbf{a}} \,, \\
\mathcal{J}_{\phi_1}^\alpha &= \frac{\mathcal{L}_{NG}}{\partial_\alpha\phi_1} = -\frac{1}{2\pi\alpha'}g_{\phi_1\phi_1}\partial_\beta\phi_1\left(\mathbf{a}^{-1}\right)^{\beta\alpha}\sqrt{-\det\mathbf{a}} - \frac{1}{2\pi\alpha'}\varepsilon^{\alpha\beta}b_{\phi_1\phi_2}\partial_\beta\phi_2 \,, \\
\mathcal{J}_{\phi_2}^\alpha &= \frac{\mathcal{L}_{NG}}{\partial_\alpha\phi_2} = -\frac{1}{2\pi\alpha'}g_{\phi_2\phi_2}\partial_\beta\phi_2\left(\mathbf{a}^{-1}\right)^{\beta\alpha}\sqrt{-\det\mathbf{a}} - \frac{1}{2\pi\alpha'}\varepsilon^{\alpha\beta}b_{\phi_2\phi_1}\partial_\beta\phi_1 \ .
\end{aligned} \tag{B.9}$$

Then for the ansatz (B.5) we obtain following form of conserved charges P_t, J_1, J_2

$$\begin{aligned}
P_t &= -\frac{R^2\kappa}{2\pi\alpha'}\int_0^{2\pi} d\sigma\frac{\mathbf{a}_{\sigma\sigma}}{\sqrt{-\det\mathbf{a}}} \,, \\
J_1 &= \frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma\frac{R^2G\sin^2\theta}{\sqrt{-\det\mathbf{a}}}[(\omega_1 + \beta\tilde{\phi}_1')\mathbf{a}_{\sigma\sigma} - \alpha\tilde{\phi}_1'\mathbf{a}_{\tau\sigma}] - \\
&-\frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma R^2\tilde{\gamma}\alpha G\sin^2\theta\cos^2\theta\tilde{\phi}_2' \,, \\
J_2 &= \frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma\frac{R^2G\cos^2\theta}{\sqrt{-\det\mathbf{a}}}[(\omega_2 + \beta\tilde{\phi}_2')\mathbf{a}_{\sigma\sigma} - \alpha\tilde{\phi}_2'\mathbf{a}_{\tau\sigma}] + \\
&+\frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma R^2\tilde{\gamma}\alpha G\sin^2\theta\cos^2\theta\tilde{\phi}_1' \ .
\end{aligned} \tag{B.10}$$

Nambu-Goto action is still diffeomorphism invariant. We fix this gauge freedom by demanding that

$$\sqrt{-\det\mathbf{a}} = \sqrt{-\mathbf{a}_{\tau\tau}\mathbf{a}_{\sigma\sigma} + \mathbf{a}_{\tau\sigma}^2} = \mathbf{a}_{\sigma\sigma} \tag{B.11}$$

that can be solved with the condition[8]

$$\mathbf{a}_{\tau\tau} = 0 \tag{B.12}$$

[8]It is clear that we could use an alternative gauge fixing solution $\mathbf{a}_{\tau\tau} = -\mathbf{a}_{\sigma\sigma}\,, \mathbf{a}_{\tau\sigma} = 0$.

and hence

$$\mathbf{a}_{\sigma\sigma} = \mathbf{a}_{\tau\sigma} \ , \quad \sqrt{-\det \mathbf{a}} = \mathbf{a}_{\tau\sigma} \ . \tag{B.13}$$

With this gauge fixing the charges given in (B.10) are equal to

$$\begin{aligned}
P_t &= -\frac{R^2\kappa}{2\pi\alpha'}\int_0^{2\pi} d\sigma \ , \\
J_1 &= \frac{R^2}{2\pi\alpha'}\int_0^{2\pi} d\sigma G \sin^2\theta \left[\omega_1 - \tilde{\phi}_1'\frac{a}{\kappa} - \alpha\tilde{\gamma}\cos^2\theta\tilde{\phi}_2'\right] \\
J_2 &= \frac{R^2}{2\pi\alpha'}\int_0^{2\pi} d\sigma G \cos^2\theta \left[\omega_2 - \tilde{\phi}_2'\frac{a}{\kappa} + \alpha\tilde{\gamma}\sin^2\theta\tilde{\phi}_1'\right] \ .
\end{aligned} \tag{B.14}$$

Then using (B.12) and (B.13) it is easy to see that the equation of motion for ϕ_1 implies following differential equation for $\tilde{\phi}_1'$

$$\tilde{\phi}_1' = \frac{1}{\beta^2 - 2\beta\alpha}\left[\frac{b}{R^2 G\sin^2\theta} - \alpha\omega_2\tilde{\gamma}\cos^2\theta - \omega_1(\beta-\alpha)\right] \ , \tag{B.15}$$

where b is constant. Note that if we choose the parametrisation $\beta - \alpha = -\beta'$ we obtain

$$\tilde{\phi}_1' = \frac{1}{(\beta'^2 - \alpha^2)}\left[\frac{b}{R^2 G\sin^2\theta} - \alpha\omega_2\gamma\cos^2\theta + \omega_1\beta_1'\right] \tag{B.16}$$

that coincides exactly with the equations of motion given in [33]. In the same way the equation of motion for ϕ_2 implies

$$\tilde{\phi}_2' = \frac{1}{\beta^2 - 2\beta\alpha}\left[\frac{c}{R^2 G\cos^2\theta} - \omega_2(\beta-\alpha) + \alpha\omega_1\tilde{\gamma}\sin^2\theta\right] \ , \tag{B.17}$$

where c is again constant.

In order to find differential equation for θ we use the condition $\mathbf{a}_{\tau\tau} = 0$ together with (B.15) and (B.17) and we obtain

$$\begin{aligned}
\theta'^2 = \frac{1}{(\beta^2 - 2\beta\alpha)^2}\Big[&\kappa^2(\beta^2 - 2\beta\alpha + 2\alpha^2) - \frac{b^2}{R^4 G\sin^2\theta} - \frac{c^2}{R^4 G\cos^2\theta} + \\
&+\frac{2\tilde{\gamma}\alpha}{R^2}(\omega_2 b\cos^2\theta - \omega_1 c\sin^2\theta) - \alpha^2(\omega_1^2\sin^2\theta + \omega_2^2\cos^2\theta)\Big] \ ,
\end{aligned} \tag{B.18}$$

where we have also used the relation

$$0 = \kappa^2 + \frac{\omega_2 c}{R^2(\alpha-\beta)} + \frac{\omega_1 b}{R^2(\alpha-\beta)} \ . \tag{B.19}$$

This relation follows from the condition $\mathbf{a}_{\tau\sigma} = \mathbf{a}_{\sigma\sigma}$ that implies

$$\theta'^2(\alpha\beta - \alpha^2) + \alpha\omega_1 G\sin^2\theta\tilde{\phi}_1' + \alpha\omega_2 G\cos^2\theta\tilde{\phi}_2' = (\alpha^2 - \alpha\beta)[G\sin^2\theta\tilde{\phi}_1'^2 + G\cos^2\theta\tilde{\phi}_2'^2] \tag{B.20}$$

Then if we combine this result with the condition $\mathbf{a}_{\tau\tau} = 0$ and use (B.15) and (B.17) we finally obtain (B.19).

It is easy to see that if we make the substitution $\beta = \alpha - \beta'$ in (B.18) we obtain the same equation that was presented in [33]. A careful analysis presented there shows that there exist two solutions corresponding to giant magnon and spikes. We will not repeat these calculations here and recommend the original paper [33] for more details.

C. Conserved charges for the D1-brane in β deformed background

The conserved charges P_t, J_1, J_2 for the D1-brane in the β deformed background can be calculated as

$$\begin{aligned}
P_t &= -\tau_1 R^2 \kappa \int_0^{2\pi} d\sigma \frac{e^{-\Phi_0}}{\sqrt{G}} \sqrt{1+e^{2\Phi_0} G\Pi^2} \,, \\
J_1 &= \tau_1 R^2 \int_0^{2\pi} d\sigma [e^{-\Phi_0}\sqrt{G}\sin^2\theta(\omega_1+(\beta-\alpha)\tilde{\phi}_1')\sqrt{1+e^{2\Phi_0}G\Pi^2} - \\
&\quad -\tilde{\gamma} G\cos^2\theta\sin^2\theta\alpha\tilde{\phi}_2'\Pi] \,, \\
J_2 &= \tau_1 R^2 \int_0^{2\pi} d\sigma [e^{-\Phi_0}\sqrt{G}\cos^2\theta(\omega_2+(\beta-\alpha)\tilde{\phi}_2')\sqrt{1+e^{2\Phi_0}G\Pi^2} + \\
&\quad +\tilde{\gamma} G\cos^2\theta\sin^2\theta\alpha\tilde{\phi}_1'\Pi] \,.
\end{aligned} \tag{C.1}$$

Further, the equations of motion for ϕ_1 reduces to

$$\begin{aligned}
\tilde{\phi}_1' &= \frac{1}{\sqrt{1+e^{2\Phi_0}G\Pi^2}(2\alpha\beta-\beta^2)} \times \\
&\times [\frac{Be^{\Phi_0}}{R^2\sqrt{G}\sin^2\theta} - \alpha\omega_2\Pi e^{\Phi_0}\tilde{\gamma}\sqrt{G}\cos^2\theta + (\beta\omega_1-\alpha\omega_1)\sqrt{1+e^{2\Phi_0}G\Pi^2}] \,.
\end{aligned} \tag{C.2}$$

In the same way the equation of motion for ϕ_2 gives

$$\begin{aligned}
\tilde{\phi}_2' &= \frac{1}{\sqrt{1+e^{2\Phi_0}G\Pi^2}(2\alpha\beta-\beta^2)} \times \\
&\times [\frac{Ce^{\Phi_0}}{R^2\sqrt{G}\cos^2\theta} + \alpha\omega_1\Pi e^{\Phi_0}\tilde{\gamma}\sqrt{G}\sin^2\theta + (\beta\omega_2-\alpha\omega_2)\sqrt{1+e^{2\Phi_0}G\Pi^2}]
\end{aligned} \tag{C.3}$$

Now if we combine the diffeomorphism invariance condition (4.2) together with the condition $\mathbf{A}_{\tau\tau} = 0$ we obtain the relation

$$0 = \kappa^2 - \frac{\sqrt{G}e^{\Phi_0}}{(\alpha-\beta)\sqrt{1+e^{2\Phi_0}G\Pi^2}R^2}[B\omega_1 + C\omega_2] \,. \tag{C.4}$$

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